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determination of vapor bubble breakabay dimensions in hich-speed flows
V. A. Gerliga, A. V. Korolev,

UDC 536.423.1:532.517
and V. I. Skalozubov

A semiempirical relationship is proposed for determining the diameter at which vapor bubbles break away from a channel wall in high-speed flows of boiling liquids.

A large number of experimental and theoretical studies have been published to date concerning the determination of bubble breakaway diameter in liquid boiling on a heating surface under natural convection conditions. For example, a detailed review of the state of this problem can be found in [1, 2]. However, experimental studies under conditions of forced flow motion [3, 4] have produced qualitative and quantitative divergence from the results of the above studies. Thus, under certain conditions the bubble breakaway diameter is an order of magnitude or more lower than under corresponding natural convection conditions. The limited applicability of the few [3, 5] empirical descriptions is related to the insufficient volume of experimental data and the relatively narrow parameter ranges studied.

In connection with this fact, analytical studies are of definite interest. Unfortunately, to date the number of theoretical studies on determination of bubble breakaway diameter has been quite limited. The complexity of this problem is related foremost to the large number of factors affecting bubble breakaway conditions. Second, analysis of presently available studies, for example [6-9] et al., has shown that there is a diversity of opinion as to definition of the magnitude, direction, and character of the action of some forces. Moreover, in many studies the conditions used to define the moment of bubble breakaway from the wall are not well justified.

In considering the major factors affecting the breakaway of bubbles under conditions of both natural and forced convection the majority of authors agree that the main forces supporting bubbles during the breakaway process are forces produced by liquid relaxation in response to bubble growth and surface tension forces. Thus one can distinguish dynamic ( $\mathrm{F}_{\mathrm{R}} \gg$ $F_{\sigma}$ ) and quasistatic ( $F_{\sigma} \gg F_{R}$ ) breakaway regimes according to [10].

Below we will consider the problem of vapor bubble breakaway in high-speed flows under quasistatic breakaway regime conditions at relatively low superheating levels. According to the results of [7] the force $F_{R}$ may be neglected at $J_{a}<10$. Low superheat levels define a relatively low value of vapor formation center density. Therefore the effect of bubble interaction on breakaway will not be considered.

It should be noted that at present there is not a generally accepted definition of the value of the force which compensates surface tension $\mathrm{F}_{\sigma}$ under such conditions. Thus, in [11, 12] the authors proposed a definition of the surface tension force at the moment of breakaway in the form

$$
\begin{equation*}
F_{\sigma}=2 \pi R_{c} \sigma, \tag{1}
\end{equation*}
$$

where $R_{C}$ is the radius of the microfissure which serves as the vapor formation center. The quantity $R_{C}$ then corresponds to the radius of the critical vapor bubble nucleus.

[^0]In [4, 7] another expression was proposed for the surface tension force

$$
\begin{equation*}
F_{\mathrm{o}}=2 \pi R \sigma \sin \theta_{\mathrm{o}} \tag{2}
\end{equation*}
$$

where $R$, $\theta_{0}$ are some mean values of bubble radius and dynamic contact angle. It follows from the expressions presented that the values of the surface tension force defined by Eqs. (1) and (2) may differ greatly from each other.

The force which compensates the surface tension $F_{\sigma}$ arises as a result of change in the relative direction of stresses on the bubble surface, which is produced by some external action (i.e., forces which tend to remove the bubble from the solid surface). The direction of the force $F_{\sigma}$ will then be determined by the direction of the resulting external action (Fig. 1).

It can be shown that for the case where the bubble contact line extends beyond the limits of the microfissure, the expression for the force which compensates surface tension $F_{\sigma}$ has the form:

$$
\begin{equation*}
F_{\sigma}=\oint_{l} \sigma\left[\cos \left(\psi-\theta_{2}(l)\right)-\cos \left(\theta_{1}(l)+\psi\right)\right] d l \tag{3}
\end{equation*}
$$

where $\theta_{1}(Z), \theta_{2}(Z)$ are the "forward" and "rear" contact angles with respect to the direction of the incident flow, varying along the line of contact.

Under conditions where the hydrodynamic resistance force $F_{W}$ dominates over other external effects $(\psi=0)$ the authors of [8] defined the force $F_{\sigma}$ in the form

$$
\begin{equation*}
F_{\sigma}=\frac{1}{2} \pi \sigma R \sin \theta_{0}\left[\cos \theta_{2}-\cos \theta_{1}\right] \tag{4}
\end{equation*}
$$

where $\theta_{0}$ is the equilibrium contact angle, and the quantity $R \sin \theta_{0}$ characterizes the mean length of the line of contact between bubble and surface.

A long series of experiments performed in [8] revealed some disagreement between calculated and experimental data when Eq. (4) was used. The authors explained this divergence as being due to departure of the shape of the contact line between bubble and solid surface from circular form and introduced an empirical correction function dependent on $\theta_{0}$. However, the value of $\theta_{0}$ was defined in [8] for each type of liquid and channel surface before performing the main experiments, using the equilibrium position of a drop on the channel surface. Generally speaking, this value of $F_{\sigma}$ may differ from the mean value of the dynamic contact angle during the bubble breakaway process.

It should be noted that at the moment of breakaway the bubble surface is surrounded by liquid only. Consequently, according to Eq. (3) the force compensating $\mathrm{F}_{\sigma}$ should be equal to zero. Therefore Eqs. (1), (2), (4) do not reflect the true value of the force compensating surface tension $F_{\sigma}$ directly at the moment of breakaway, since according to Eqs. (1), (2), (4) the force compensating $F_{\sigma}$ is nonzero, which contradicts the physical situation.

Considering Eq. (3) we will define $F_{\sigma}$ with the expression

$$
\begin{equation*}
F_{\sigma}=\frac{1}{2} \pi \sigma R f\left(\theta, \theta_{1}, \theta_{2}, \psi\right) \tag{5}
\end{equation*}
$$

where

$$
f\left(\theta, \theta_{1}, \theta_{2}, \psi\right)=\sin \theta\left[\cos \left(\psi-\theta_{2}\right)-\cos \left(\psi+\theta_{1}\right)\right]
$$

The quantity $\theta$ is some mean contact angle between bubble and surface. It should then be noted that the during the process of growth on the solid surface the bubble has some irregular form, being extended in the direction of the external force (Fig. 1). Therefore, by the term radius we will understand the radius of a normalized bubble, having the form of a truncated sphere, the volume of which is equal to the volume of the real bubble. Then the value of the contact angle $\theta$ of the normalized bubble is chosen such that the force $\mathrm{F}_{\sigma}$ acting on the normalized bubble will correspond to the same force acting on the real bubble.

In defining the value of the hydrodynamic resistance force $\mathrm{F}_{\mathrm{W}}$ it is important to clarify the character of motion of the liquid flow incident on the bubble. Some authors, in particular, those of [4], have assumed that the bubble breakaway diameter is comparable to the thickness of the laminar sublayer. However, approximate calculations with consideration of available experimental results [3, 8, 14] have confirmed the opinion of Blinkov [11] that the


Fig. 1


Fig. 2

Fig. 1. Computation of vapor bubble breakaway from channel surface under high-speed boiling flow conditions.
Fig. 2. Comparison of Eq. (11) with experimental data of other authors: 1) [4]; 2) [14]; 3) [12]; 4) [8]; 5) [3]; $A=(\lambda / 8)^{-1} \times$ $(\sigma / \rho Z)^{7 / 9} v^{2 / 9} W^{-16 / 9}, m ; d_{b r}, m$.
breakaway dimension significantly exceeds the thickness of the laminar sublayer. In this case the mean velocity of the incident flow can be defined from the well-known law of velocity distribution (for $\operatorname{Re}>10^{4}$ ) over thickness of a turbulent boundary layer [2]

$$
\begin{equation*}
w=8.74\left(\frac{\lambda}{8}\right)^{\frac{9}{14}} W^{\frac{8}{7}} R^{\frac{1}{7}} v^{-\frac{1}{7}} . \tag{6}
\end{equation*}
$$

Consequently, we may write $\mathrm{F}_{\mathrm{w}}$ in the form

$$
\begin{equation*}
F_{w}=C_{1}\left(\frac{\lambda}{8}\right)^{\frac{9}{7}} \pi \rho_{l} \frac{W^{\frac{16}{7}} R^{\frac{16}{7}}}{v^{\frac{2}{7}}} \tag{7}
\end{equation*}
$$

Upon satizfaction of the condition

$$
\begin{equation*}
\frac{10 \rho_{l}\left(\frac{\lambda}{8}\right)^{\frac{9}{7}} W^{\frac{16}{7}}}{\left(\rho_{l}-\rho_{v}\right) g v^{\frac{2}{7}}} \gg 1 \tag{8}
\end{equation*}
$$

the effect of gravitation $\mathrm{F}_{\mathrm{g}}$ may be neglected in comparison to $\mathrm{F}_{\mathrm{W}}$.
An important question in the problem under consideration is formulation of the conditions corresponding to the moment of bubble breakaway from the surface. Labuntsov [13] justifiably criticized the proposition that equilibrium of forces is a necessary and sufficient condition for breakaway. Despite this fact, the overwhelming majority of studies (for example, $[4,6-9,11,12]$ etc.) have identified the moment of breakaway with equilibrium of the foces acting on the bubble, which under certain conditions (for example, at a high bubble growth rate) can lead to a significant difference in breakaway size from the corresponding value at the moment of force equilibrium.

The most general approach to the process of breakaway in a dynamic regime ( $\mathrm{F}_{\mathrm{R}} \gg \mathrm{F}_{\sigma}$ ) under conditions where the flow has no effect on bubbles on the wall was considered by Kirichenko [10]. In contrast to [13], he assumed that in the initial stage of bubble growth the forces restraining the bubble exceed those tending to remove it from the wall. In Kirichenko's opinion, in the general case bubble breakaway occurs when equalization of forces sets in, at the moment when the center of mass of the bubble $S$ is located at some distance from the solid surface, related to the breakaway dimension by the expression

$$
\begin{equation*}
S_{\mathrm{br}}=k R_{\mathrm{br}} \tag{9}
\end{equation*}
$$

where $k$ is an empirical constant $(k \simeq 1.5)$. Breakaway conditions must be determined from the nature of the dominant forces and the character of their action on the bubble over the entire growth period up to the moment of breakaway.

Now in the quasistatic regime considered herein should we regard equilibrium of forces (in particular, $F_{\sigma}=F_{W}$ ) as a necessary and sufficient condition for breakaway. In fact, as
was indicated above, at the moment of breakaway the main force restraining the bubble $F_{\sigma}$ is equal to zero, while the compensating force tending to removal the bubble from the wall is nonzero.

In the process of bubble growth $F_{\sigma}$ is compensated by an increasing external effect tending to break the bubble away. Such an equilibrium of forces will be found up to some moment (which we will term the critical moment), after which the state of the bubble on the wall becomes unstable. This means that at subsequent times the force $F_{\sigma}$ cannot compensate the increasing external effect. At the moment of critical equilibrium $F_{\sigma}$ attains some critical value ( $\mathrm{F}_{\sigma}$ ) cr. However, even if most coarse assumptions are made, derivation of an analytical expression for the quantity $f$ (and hence, $F_{\sigma}$ ) at the moment of critical equilibrium is extremely complicated for a number of reasons. At this time the quantity $f$ is determined mainly by the surface structure and nature of the contacting phases. Therefore for water and approximately identically processed surfaces (without special processing) as a first approximation we will take the quantity $f$ equal to some constant.

The mean bubble radius at the moment of critical equilibrium for $\left(F_{\sigma}\right)_{c r}=F_{\mathrm{w}}$ can be defined by the expression

$$
\begin{equation*}
R_{\mathrm{cr}}=\left(\frac{f \mathrm{cr}}{2 C_{1}}\right)^{\frac{7}{9}}\left(\frac{\lambda}{8}\right)^{-1}\left(\frac{\sigma}{\rho_{l}}\right)^{\frac{7}{9}} v^{\frac{2}{9}} W^{-\frac{16}{9}} . \tag{10}
\end{equation*}
$$

At subsequent times after critical equilibrium has been attained the stage of bubble breakaway from the solid surface takes place. This stage continues as long as the bubble surface is not completely surrounded by liquid. At the moment of breakaway, as was indicated above, $\left(\mathrm{F}_{\sigma}\right)_{\mathrm{br}}=0$. Consequently, the necessary condition for breakaway may be taken as $\mathrm{f}=$ $\mathrm{f}_{\mathrm{cr}}$, while the sufficient condition is $\mathrm{f}_{\mathrm{br}}=U$.

Determination of the ratio between the length of the breakaway stage and the length of the bubble growth stage before critical equilibrium is attained, just like determination of $\mathrm{f}_{\mathrm{cr}}$, is extremely complicated in the general case. Therefore, we will assume that breakaway takes place immediately after the bubble enters the unstable state corresponding to critical equilibrium.

Thus, considering the above, as well as Eq. (10), we propose that the breakaway dimensions for vapor bubbles in high-speed flows under quasistatic conditions may be defined by the expression

$$
\begin{equation*}
\frac{d_{\mathrm{br}}}{D}=C_{0}\left(\frac{\lambda}{8}\right)^{-1} \mathrm{We}^{-\frac{7}{9}} \mathrm{Re}^{-\frac{2}{9}}, \tag{11}
\end{equation*}
$$

where $C_{0}=\left(\mathrm{f}_{\mathrm{Cr}} / \mathrm{ZC}_{1}\right)^{7 / 9}$ (for water and solid surfaces without special preparation) -a value which is constant and can be estimated by comparison with corresponding experiments.

To determine $C_{0}$ known experimental data were used (in the parameter range $W \leqslant 10 \mathrm{~m} / \mathrm{sec}$, $P \leqslant 4.0 \mathrm{MPa}$ ), obtained upon breakaway of bubbles from a water heating surface under forced convection conditions [3, 4, 14], as well as for gas bubble breakaway in high-speed water flows [8, 12]. Estimates of the major forces acting upon bubbles on the wall showed that conditions in all the experiments indicated corresponded to the quasistatic breakaway regime ( $\mathrm{Ja}<10$ ). Equation (11) agrees satisfactorily with the data of $[3,4,12,14]$ (Fig. 2). On this basis, we find $C_{0}=1.1 \cdot 10^{-2}$. At such a value of $C_{0}$ there is quantitative (but not qualitative) disagreement with the data of [8], obtained with specially processed glass surfaces and addition of surface active materials. However, those experimental data confirm that the critical value $\left(\mathrm{F}_{\sigma}\right)_{\mathrm{cr}}$ or $\left(\mathrm{f}_{\mathrm{cr}}\right)$ depends on the channel surface structure. The cause of the divergence of the results of [8] from those of [3, 4, 12, 14], obtained on metal surfaces without special processing, becomes obvious. The data of [8] do agree well with Eq. (11) qualitatively.

An expression similar to Eq. (11) was obtained in [9] to describe breakaway dimensions of vapor bubbles from a channel surface under forced convection conditions:

$$
\begin{equation*}
\frac{\mathrm{d}_{\mathrm{br}}}{D}=(2-3.8) \mathrm{We}^{-\frac{7}{9}} . \tag{12}
\end{equation*}
$$

Equation (12) agrees well with the results of [4]. However it should be noted that the experimental data of [4] were obtained with relatively low flow velocities ( $\mathrm{W}<1 \mathrm{~m} / \mathrm{sec}$ ). In
defining Eq. (11) the only experimental data from [4] used were those obtained at $W \geqslant 0.3 \mathrm{~m} /$ sec. At lower flow velocities the results agree poorly with Eq. (11), since apparently Eq. (8) is not satisfied.

Thus, as compared to available recommendations for determination of vapor bubble breakaway diameter under quasistatic conditions (Ja < 10) , Eq. (11) is applicable over a wider parameter range ( $1.0 \leqslant P \leqslant 4.0 \mathrm{MPa}, 0.3 \leqslant W \leqslant 10 \mathrm{~m} / \mathrm{sec}$ ).

## NOTATION

$\mathrm{D}_{\mathrm{b}}, \mathrm{R}_{\mathrm{b}} \mathrm{r}$, diameter and radius of bubble, m ; D , channel diameter, m ; W , mean flow velocity over channe1 section, $\mathrm{m} / \mathrm{sec}$; We $=\rho D W^{2} / \sigma$, Weber number; $\mathrm{Re}=\mathrm{WD} / \nu$, Reynolds number; $\lambda$, friction coefficient; $\mathrm{F}_{\mathrm{w}}, \mathrm{Fg}_{\mathrm{g}}, \mathrm{F}_{\delta}, \mathrm{F}_{\mathrm{R}}$, hydrodynamic resistance, withdrawal, surface tension, and liquid response forces, respectively, $\mathrm{N} ; \mathrm{k}, \mathrm{C}_{1}, \mathrm{C}_{0}$, dimensionless constants; $\theta$, contact angle, rad; $\sigma$, surface tension coefficient, $\mathrm{N} / \mathrm{m} ; \rho_{\mathcal{I}}, \rho_{\mathrm{V}}$, liquid and vapor densities, $\mathrm{kg} / \mathrm{m}^{3} ; \mathrm{J}_{a}$, Jacobi number; $l$, length of bubble base, $m ; \varphi$, angle of channel inclination to the horizontal, rad; $\psi$, angle of action of resultant external force, rad; w, mean incident flow velocity over bubble height, $\mathrm{m} / \mathrm{sec} ; \mathrm{g}$, acceleration of gravity, $\mathrm{m} / \mathrm{sec}^{2} ; ~ v$, kinematic viscosity of liquid, $\mathrm{m}^{2} / \mathrm{sec} ; \theta_{1}, \theta_{2}$, forward and rear contact angles relative to incident $f 1 \mathrm{ow}$, rad; $\tau$, time, sec. Subscripts: 0, equilibrium value; cr, critical equilibrium; br, breakaway.

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